# Solution Key Mathematics B – Practice exam 2

*The texts printed in grey are given to clarify the steps taken to obtain a given solution. To obtain the given scores, only the text printed in black (or equivalent formulations) need to be given.*

## Subject-specific marking rules and guidelines

**1.** For each error or mistake in calculation or notation a single point will be subtracted from the maximum score that can be obtained for that particular part of the question.

**2.** If a required explanation, deduction or calculation has been omitted or has been stated incorrectly 0 points will be awarded, unless otherwise stated in the solution key. This is also the case for answers obtained by the use of a graphic calculator. Answers obtained by the graphic calculator should indicate how the graphic calculator has been used to obtain the answer. Candidates must make sure they mention formulas applied or provide lists and calculation methods used in their answers.

**3.** A particular mistake in the answer to a particular exam question will lead to a deduction of points only once, unless the question is substantially simplified by the mistake and/or when the solution key specifies otherwise.

**4.** A repeated mistake made in the answer to different exam questions will lead to a deduction of points each time such a mistake has been made, unless the solution key specifies otherwise.

**5.** If only one example, reason, explication, explanation or any other type of answer is required and more than one has been given, only the first answer given will be graded. If more than one example, reason, explication, explanation or any other type of answer is required, only the first answers are graded, up to and including the number of answers specified by the exam question.

**6.** If the candidate fails to give a required unit in the answer to a question a single point will be subtracted from the total score, unless the unit has been specified in the exam question.

**7.** If during intermediate steps results are rounded, resulting in an answer different from one in which non-rounded intermediate results are used, one point will be subtracted from the total score. Rounded intermediate results may, however, be noted down.

**Question 1.**

| | | |
|---|---|---|
| a. | *Alternative 1:*<br>$f'(x) = \frac{1}{2\sqrt{4x+8}} \cdot 4 = \frac{2}{\sqrt{4x+8}}$ | 2 |
| | $f'(-1) = \frac{2}{\sqrt{-4+8}} = 1$ | 1 |
| | Substituting $A(-1,2)$ into $y = x + b$ gives $2 = -1 + b$ and therefore $b = 3$ | 1 |
| | Reducing $y = x + 3$ to zero gives $y - x - 3 = 0$ | 1 |
| | *Alternative 2:*<br>$f'(x) = \frac{1}{2\sqrt{4x+8}} \cdot 4 = \frac{2}{\sqrt{4x+8}}$ | 2 |
| | $f'(-1) = \frac{2}{\sqrt{-4+8}} = 1$ | 1 |
| | $slope_l = -\frac{a}{b} = -\frac{-1}{1} = 1$ (so the slope of the graph of $f$ at point $A$ equals the slope of line $l$.) | 1 |
| | Substituting $A(-1,2)$ into line $l$ gives : $2 - -1 - 3 = 0$. (So $A$ lies on line $l$.) | 1 |
| b. | *Alternative 1:*<br>(Left boundary:) Setting $y = 0$ for line $l$ gives $0 - x - 3 = 0$ so $x = -3$ | 1 |
| | (Middle boundary:) Solving $f(x) = 0$ gives $4x + 8 = 0$ so $x = -2$ | 1 |
| | $A_V = \int_{-3}^{-1} (x+3)\,\mathrm{d}x - \int_{-2}^{-1} \sqrt{4x+8}\,\mathrm{d}x = \int_{-3}^{-1} (x+3)\,\mathrm{d}x - \int_{-2}^{-1} (4x+8)^{\frac{1}{2}}\,\mathrm{d}x$ | 1 |
| | $A_V = \left[\frac{1}{2}x^2 + 3x\right]_{-3}^{-1} - \left[\frac{1}{4} \cdot \frac{1}{1\frac{1}{2}} \cdot (4x+8)^{1\frac{1}{2}}\right]_{-2}^{-1} \left(= 2 - \left[\frac{1}{6}(4x+8)\sqrt{4x+8}\right]_{-2}^{-1}\right)$ | 2 |
| | $A_V = 2 - \left(\frac{1}{6} \cdot 4\sqrt{4} - 0\right) = 2 - \frac{4}{3} = \frac{2}{3}$ | 1 |
| | *Alternative 2:*<br>(Left boundary:) Setting $y = 0$ for line $l$ gives: $0 - x - 3 = 0$ so $x = -3$ | 1 |
| | (Middle boundary:) Solving $f(x) = 0$ gives $4x + 8 = 0$ so $x = -2$ | 1 |
| | $A_V = \text{Area triangle} - \int_{-2}^{-1} \sqrt{4x+8}\,\mathrm{d}x = 2 - \int_{-2}^{-1} (4x+8)^{\frac{1}{2}}\,\mathrm{d}x$ | 1 |
| | $\text{Area triangle} = \frac{1}{2} \cdot (x_A - -3) \cdot y_A = \frac{1}{2} \cdot 2 \cdot 2$ | 1 |
| | $A_V = 2 - \left[\frac{1}{4} \cdot \frac{1}{1\frac{1}{2}} \cdot (4x+8)^{1\frac{1}{2}}\right]_{-2}^{-1} \left(= 2 - \left[\frac{1}{6}(4x+8)\sqrt{4x+8}\right]_{-2}^{-1}\right)$ | 1 |
| | $A_V = 2 - \left(\frac{1}{6} \cdot 4\sqrt{4} - 0\right) = 2 - \frac{4}{3} = \frac{2}{3}$ | 1 |
| | *Alternative 3:*<br>(Left boundary:) Setting $y = 0$ for line $l$ gives $0 - x - 3 = 0$ so $x = -3$ | 1 |
| | (Middle boundary:) Solving $f(x) = 0$ gives $4x + 8 = 0$ so $x = -2$ | 1 |
| | $A_V = \int_{-3}^{-2} (x+3)\,\mathrm{d}x + \int_{-2}^{-1} \left(x + 3 - \sqrt{4x+8}\,\right)\mathrm{d}x$<br>$= \int_{-3}^{-2} (x+3)\,\mathrm{d}x + \int_{-2}^{-1} \left(x + 3 - (4x+8)^{\frac{1}{2}}\right)\mathrm{d}x$ | 1 |
| | $A_V = \left[\frac{1}{2}x^2 + 3x\right]_{-3}^{-2} + \left[\frac{1}{2}x^2 + 3x - \frac{1}{4} \cdot \frac{1}{1\frac{1}{2}} \cdot (4x+8)^{1\frac{1}{2}}\right]_{-2}^{-1}$ | 2 |
| | $A_V = \frac{1}{2} + 1\frac{1}{2} - \frac{1}{6} \cdot 4\sqrt{4} - 0 = 2 - \frac{4}{3} = \frac{2}{3}$ | 1 |

**Question 2.**

| | | |
|---|---|---|
| a. | *Alternative 1:*<br>The equation $f(x) = \lvert x^3 - 3x^2 \rvert = 2x$ gives $x^3 - 3x^2 = 2x \vee x^3 - 3x^2 = -2x$ | 1 |
| | Reducing $x^3 - 3x^2 = 2x$ to $x(x^2 - 3x - 2) = 0$ | 1 |
| | The solutions are: $x = 0 \vee x = \frac{3+\sqrt{17}}{2} \vee x = \frac{3-\sqrt{17}}{2}$ | 1 |
| | Reducing $x^3 - 3x^2 = -2x$ to $x(x-1)(x-2) = 0$ | 1 |
| | The solutions are: $x = 0 \vee x = 1 \vee x = 2$ | 1 |
| | $x = \frac{3-\sqrt{17}}{2} < 3$, so $x = \frac{3-\sqrt{17}}{2}$ is not a solution. So there are 4 common points | 1 |
| | *Alternative 2:*<br>$f(x) = x^3 - 3x^2$ if $x \geq 3$ (or if $x = 0 \vee x \geq 3$)<br>$f(x) = -x^3 + 3x^2$ if $x < 3$ (or if $x < 0 \vee 0 < x < 3$) | 1 |
| | Reducing $x^3 - 3x^2 = 2x$ to $x(x^2 - 3x - 2) = 0$ | 1 |
| | Solutions: $x = 0 \vee x = \frac{3+\sqrt{17}}{2} \vee x = \frac{3-\sqrt{17}}{2}$ | 1 |
| | Rewriting $-x^3 + 3x^2 = 2x$ to $x(x-1)(x-2) = 0$ | 1 |
| | Solutions: $x = 0 \vee x = 1 \vee x = 2$ | 1 |
| | $x = \frac{3-\sqrt{17}}{2} < 3$, so $x = \frac{3-\sqrt{17}}{2}$ is not a solution. So there are 4 common points | 1 |
| b. | Boundaries: $f(x) = 0$ gives $x^2(x-3) = 0$ | 1 |
| | Solutions: $x = 0 \vee x = 3$ | 1 |
| | $V = \pi \int_0^3 (f(x)^2) dx = \pi \int_0^3 (x^3 - 3x^2)^2 \, \mathrm{d}x$ (of $V = \pi \int_0^3 (-x^3 + 3x^2)^2 \, \mathrm{d}x$) | 1 |
| | $V = \pi \int_0^3 (x^6 - 6x^5 + 9x^4) \, \mathrm{d}x$ | 1 |
| | $V = \pi \left[\frac{1}{7}x^7 - x^6 + \frac{9}{5}x^5\right]_0^3$ | 1 |
| | $V = 20\frac{29}{35}\pi \approx 65{,}43$ | 1 |

**Question 3.**

| | | |
|---|---|---|
| a. | *Alternative 1:*<br>$\overrightarrow{HA} = \vec{a} - \vec{h} = \binom{0}{0} - \binom{8}{4} = \binom{-8}{-4}$<br>$\overrightarrow{HB} = \vec{b} - \vec{h} = \binom{14}{2} - \binom{8}{4} = \binom{6}{-2}$ | 1 |
| | $\cos(\angle AHB) = \frac{\binom{-8}{-4} \cdot \binom{6}{-2}}{\lvert\binom{-8}{-4}\rvert \cdot \lvert\binom{6}{-2}\rvert}$ | 1 |
| | $\cos(\angle AHB) = \frac{-48+8}{\sqrt{80} \cdot \sqrt{40}} = \frac{-40}{\sqrt{3200}}$ | 1 |
| | $\cos(\angle AHB) = -\frac{1}{2}\sqrt{2}$ | 1 |
| | So: $\angle AHB = 135°$ | 1 |
| | *Alternative 2:*<br>$AB = \sqrt{14^2 + 2^2} = \sqrt{200} = 10\sqrt{2}$<br>$AH = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5}$<br>$BH = \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10}$ | 1 |
| | $AB^2 = AH^2 + BH^2 - 2 \cdot AH \cdot BH \cdot \cos(\angle AHB)$ so:<br>$200 = 80 + 40 - 2 \cdot 4\sqrt{5} \cdot 2\sqrt{10} \cdot \cos(\angle AHB)$ | 1 |
| | $\cos(\angle AHB) = -\frac{80}{16\sqrt{50}}$ | 1 |
| | $\cos(\angle AHB) = -\frac{1}{2}\sqrt{2}$ | 1 |
| | So: $\angle AHB = 135°$ | 1 |

| | | |
|---|---|---|
| b. | *Alternative 1:*<br>$\vec{n}_k = \overrightarrow{CH} = \vec{h} - \vec{c} = \binom{8}{4} - \binom{6}{18} = \binom{2}{-14}$<br>Substituting $A(0,0)$ into $2x - 14y = c$ gives $k: 2x - 14y = 0$ | 1 |
| | $\vec{n}_l = \overrightarrow{AB} = \vec{b} - \vec{a} = \binom{14}{2} - \binom{0}{0} = \binom{14}{2}$ | 1 |
| | Substituting $C(6,18)$ into $14x + 2y = c$ gives $l: 14x + 2y = 120$ | 1 |
| | Rewriting $k$ (for example) to $k: x = 7y$ and solving: $14 \cdot 7y + 2y = 120$ gives $y = 1\frac{1}{5}$ and $x = 8\frac{2}{5}$, so $D\left(8\frac{2}{5}, 1\frac{1}{5}\right)$ or $D\left(\frac{42}{5}, \frac{6}{5}\right)$ | 2 |
| | *Alternative 2:*<br>$slope_k = \frac{2-0}{14-0} = \frac{1}{7}$ (or: $k \perp l$, so $slope_k = \frac{-1}{-7} = \frac{1}{7}$)<br>Substituting $(0,0)$ into $y = \frac{1}{7}x + b$ gives $b = 0$ so: $k: y = \frac{1}{7}x$ | 1 |
| | $slope_l = \frac{4-18}{8-6} = -\frac{14}{2} = -7$ (or: $k \perp l$, so $slope_l = \frac{-1}{1/7} = -7$) | 1 |
| | Substituting $C(6,18)$ into $y = -7x + b$ gives $b = 60$, so $l: y = -7x + 60$ | 1 |
| | $\frac{1}{7}x = -7x + 60$ gives $x = 8\frac{2}{5}$ | 1 |
| | $x = 8\frac{2}{5}$ gives $y = 1\frac{1}{5}$, so $D\left(8\frac{2}{5}, 1\frac{1}{5}\right)$ or $D\left(\frac{42}{5}, \frac{6}{5}\right)$ | 1 |
| c. | *Alternative 1:*<br>Rewriting the equation of the circle into the form : $(x-6)^2 + (y-8)^2 = 100$ and noticing that the radius of circle $c$ is therefore equal to 10. | 1 |
| | Noticing that the centre $N$ of circle $d$ lies at the intersection of the perpendicular bisectors of $AB$ and $AH$ (or another pair of perpendicular bisectors). | 1 |
| | Determining an equation for the perpendicular bisectors of $AB$:<br>$\overrightarrow{AB} = \vec{b} - \vec{a} = \binom{14}{2} - \binom{0}{0} = \binom{14}{2}$ so $14x + 2y = c$. Substituting the coordinates of the centre $(7,1)$ of $AB$ (ór of $M(6,8)$) gives $14x + 2y = 100$ or $y = 50 - 7x$ | 1 |
| | Determining an equation for the perpendicular bisector of $AH$:<br>$\overrightarrow{AH} = \vec{h} - \vec{a} = \binom{8}{4} - \binom{0}{0} = \binom{8}{4}$ so $8x + 4y = c$. Centre of $AH$ is found at the tip of vector $\frac{1}{2}\left(\binom{8}{4} - \binom{0}{0}\right) = \binom{4}{2}$. Substituting $(4,2)$ gives : $8x + 4y = 40$ | 1 |
| | Determining the coordinates of the intersection of the perpendicular bisectors of $AB$ and $AH$:<br>Solving $8x + 4(50 - 7x) = 40$ gives $160 = 20x$ or $x = 8$ and $y = 50 - 7 \cdot 8 = 6$. So $N(8, -6)$. | 1 |
| | $d(A,N) = \sqrt{8^2 + (-6)^2} = 10$. Therefore the radii of the circles are equal. | 1 |
| | *Alternative 2:*<br>Rewriting the equation of the circle into the form: $(x-6)^2 + (y-8)^2 = 100$ and noticing that the radius of circle $c$ is therefore equal to 10. | 1 |
| | Equation circle $d: (x-a)^2 + (y-b)^2 = r^2$.<br>Substituting $A(0,0)$ gives: $a^2 + b^2 = r^2$ (i) | 1 |
| | Substituting $B(14,2)$ gives $(14-a)^2 + (2-b)^2 = r^2$<br>Or $196 - 28a + a^2 + 4 - 4b + b^2 = r^2$ . Together with equation (i) this yields: $200 - 28a - 4b = 0$ (ii) | 1 |
| | Substituting $H(8,4)$ gives $(8-a)^2 + (4-b)^2 = r^2$<br>Or $64 - 16a + a^2 + 16 - 8b + b^2 = r^2$. Together with equation (i) this yields: $80 - 16a - 8b = 0$ (iii) | 1 |
| | Subtracting (iii) from 2×(ii) gives : $320 - 40a = 0$ so $a = 8$ and $b = 6$ | 1 |
| | Substituting this result into (i) gives $8^2 + 6^2 = 100 = r^2$. So $r = 10$. Therefore the radii of the circles are equal. | 1 |

**Question 4.**

| | | |
|---|---|---|
| a. | Determining that $f'(x) = \frac{(x+1)\cdot[x^2]' - x^2\cdot[x+1]'}{(x+1)^2}$ | 1 |
| | $f'(x) = \frac{(x+1)\cdot 2x - x^2\cdot 1}{(x+1)^2}$ | 1 |
| | $f'(x) = \frac{2x^2+2x-x^2}{x^2+2x+1}$ | 1 |
| | $f'(x) = \frac{x^2+2x}{x^2+2x+1}$ | 1 |
| b. | It has to be the case that: $\frac{x^2}{x+1} = -3x + p \ \wedge \ \frac{x^2+2x}{x^2+2x+1} = -3$ | 1 |
| | Rewriting $\frac{x^2+2x}{x^2+2x+1} = -3$ as: $x^2 + 2x = -3x^2 - 6x - 3$ and so $4x^2 + 8x + 3 = 0$ | 1 |
| | Solutions: $x = \frac{-8 \pm \sqrt{64 - 4\cdot 4\cdot 3}}{2\cdot 4} = \frac{-8\pm\sqrt{16}}{8} = \frac{-8\pm 4}{8}$ and so $x = -1\frac{1}{2} \vee x = -\frac{1}{2}$ | 1 |
| | Analytically showing that $x = -1\frac{1}{2}$ yields $p = -9$ and $x = -\frac{1}{2}$ yields $p = -1$ | 2 |
| c. | *Alternative 1:*<br>$x^3 - x^2 = x^2(x-1)$ | 1 |
| | $x^2 - 1 = (x+1)(x-1)$ | 1 |
| | $g(x) = \frac{x^3-x^2}{x^2-1} = \frac{x^2(x-1)}{(x+1)(x-1)} = \frac{x^2}{x+1} = f(x)$ mits $x - 1 \neq 0$ ofwel $x \neq 1$ | 1 |
| | $g(1)$ $(= \frac{1-1}{1-1} = \frac{0}{0})$ is not defined. | 1 |
| | $f(1) = \frac{1^2}{1+1} = \frac{1}{2}$ so point $P\left(1, \frac{1}{2}\right)$ has been eliminated from the graph of $f$ to obtain the graph of $g$. | 1 |
| | *Alternative 2:*<br>Rewriting $\frac{x^3-x^2}{x^2-1} = \frac{x^2}{x+1}$ as $(x^3-x^2)(x+1) = x^2(x^2+1)$ for $x \neq 1 \wedge x \neq -1$ | 1 |
| | Reducing $(x^3-x^2)(x+1) = x^2(x^2+1)$ to $x^4 + x^2 = x^4 + x^2$ (or reducing it further) | 1 |
| | So $f(x) = g(x)$ for $x \neq 1$ (for $x = -1$ both functions are not defined) | 1 |
| | $g(1)$ $(= \frac{1-1}{1-1} = \frac{0}{0})$ is not defined. | 1 |
| | $f(1) = \frac{1^2}{1+1} = \frac{1}{2}$ so point $P\left(1, \frac{1}{2}\right)$ has been eliminated from the graph of $f$ to obtain the graph of $g$. | 1 |

**Question 5.**

| | | |
|---|---|---|
| a. | $y = \mathrm{e}^x$ shifted 5 units up gives $y = \mathrm{e}^x + 5$ | 1 |
| | Multiplying $y = e^x + 5$ with the factor $\frac{1}{3}$ w.r.t. the $x$-axis gives $y = \frac{1}{3}(\mathrm{e}^x + 5)$ | 1 |
| | Mirroring $y = \frac{1}{3}(\mathrm{e}^x + 5)$ in the line $y = x$ gives $x = \frac{1}{3}(\mathrm{e}^y + 5)$ | 1 |
| | Rewriting $x = \frac{1}{3}(\mathrm{e}^y + 5)$ as $3x = \mathrm{e}^y + 5$ so $\mathrm{e}^y = 3x - 5$ | 1 |
| | Rewriting $\mathrm{e}^y = 3x - 5$ as $y = \ln(3x - 5)$ | 1 |
| b. | Noticing that it has to be proved that the graph of $f$ has a point in common with line $l$ and that their slopes are equal at that point | 1 |
| | $\text{slope}_l = \frac{6}{2} = 3$ | 1 |
| | $f'(x) = \frac{1}{3x-5}\cdot 3 = \frac{3}{3x-5}$ | 1 |
| | Solving $f'(x) = \frac{3}{3x-5} = 3$ gives $3x - 5 = 1$ or $3x = 6$ so $x = 2$ | 1 |
| | $f(2) = \ln(3\cdot 2 - 5) = \ln(1) = 0$ | 1 |
| | Proving that $(2,0)$ also lies on $l$ (for example: at $t = -1 : \binom{4}{6} - \binom{2}{6} = \binom{2}{0}$) (therefore line $l$ is tangent to the graph of $f$) | 1 |

**Question 6.**

| | | |
|---|---|---|
| a. | Showing that $x_A = x_B = \frac{1}{2}\sqrt{2}$ so $\lvert AB\rvert = y_A - y_B$<br>(or: $\lvert AB\rvert = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$ | 1 |
| | The coordinates of the points in common with $x = \frac{1}{2}\sqrt{2}$ are determined by solving $\cos(t) = \frac{1}{2}\sqrt{2}$. This gives: $t = \frac{1}{4}\pi \vee t = 1\frac{3}{4}\pi$ | 1 |
| | $y_A = y\left(\frac{1}{4}\pi\right) = \sin\left(\frac{1}{2}\pi\right) + \cos\left(\frac{1}{4}\pi\right) = 1 + \frac{1}{2}\sqrt{2}$ | 1 |
| | $y_B = y\left(1\frac{3}{4}\pi\right) = \sin\left(3\frac{1}{2}\pi\right) + \cos\left(1\frac{3}{4}\pi\right) = -1 + \frac{1}{2}\sqrt{2}$ | 1 |
| | $\lvert AB\rvert = y_A - y_B = 1 + \frac{1}{2}\sqrt{2} - \left(-1 + \frac{1}{2}\sqrt{2}\right) = 2$<br>(or: $\lvert AB\rvert = \sqrt{\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}\right)^2 + \left(1 + \frac{1}{2}\sqrt{2} - \left(-1 + \frac{1}{2}\sqrt{2}\right)\right)^2} = \sqrt{0^2 + 2^2} = 2$) | 1 |
| b. | *Alternative 1:*<br>$x(t) = \cos(t) = 0$ for $t = \frac{1}{2}\pi \vee t = 1\frac{1}{2}\pi$ | 1 |
| | $\vec{v}(t) = \begin{pmatrix} -\sin(t) \\ 2\cos(2t) - \sin(t) \end{pmatrix}$ | 1 |
| | $\vec{v}_1 = \vec{v}\left(\frac{1}{2}\pi\right) = \begin{pmatrix} -\sin\left(\frac{1}{2}\pi\right) \\ 2\cos(\pi) - \sin\left(\frac{1}{2}\pi\right) \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$ | 1 |
| | $\vec{v}_2 = \vec{v}\left(1\frac{1}{2}\pi\right) = \begin{pmatrix} -\sin\left(1\frac{1}{2}\pi\right) \\ 2\cos(3\pi) - \sin\left(1\frac{1}{2}\pi\right) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ | 1 |
| | $\cos(\alpha) = \frac{\vec{v}_1 \cdot \vec{v}_2}{\lvert\vec{v}_1\rvert \cdot \lvert\vec{v}_2\rvert} = \frac{\begin{pmatrix} -1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\left\lvert\begin{pmatrix} -1 \\ -3 \end{pmatrix}\right\rvert \cdot \left\lvert\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\rvert} = \frac{2}{\sqrt{20}} = \frac{1}{\sqrt{5}}$ | 1 |
| | The requested angle is equal to $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63{,}43°$ | 1 |
| | *Alternative 2:*<br>$x(t) = \cos(t) = 0$ for $t = \frac{1}{2}\pi \vee t = 1\frac{1}{2}\pi$ | 1 |
| | $x'(t) = -\sin(t)$ and $y'(t) = 2\cos(2t) - \sin(t)$ | 1 |
| | $\left[\frac{dy}{dx}\right]_{t=\frac{1}{2}\pi} = \frac{y'\left(\frac{1}{2}\pi\right)}{x'\left(\frac{1}{2}\pi\right)} = \frac{2\cos(\pi) - \sin\left(\frac{1}{2}\pi\right)}{-\sin\left(\frac{1}{2}\pi\right)} = \frac{-3}{-1} = 3$ | 1 |
| | $\left[\frac{dy}{dx}\right]_{t=1\frac{1}{2}\pi} = \frac{y'\left(1\frac{1}{2}\pi\right)}{x'\left(1\frac{1}{2}\pi\right)} = \frac{2\cos(3\pi) - \sin\left(1\frac{1}{2}\pi\right)}{-\sin\left(1\frac{1}{2}\pi\right)} = \frac{-1}{1} = -1$ | 1 |
| | An argument showing that the requested angle is equal to<br>$180° - \left(\tan^{-1}(3) - \tan^{-1}(-1)\right)$ (of $180° - (\tan^{-1}(3) + 45°)$) | 1 |
| | Determining that the requested angle equals $180° - \left(\tan^{-1}(3) - \tan^{-1}(-1)\right) \approx 63{,}43°$ | 1 |
| c. | *Alternative 1:*<br>Rewriting $y = 2x$ as $\sin(2t) + \cos(t) = 2\cos(t)$ | 1 |
| | Rewriting the previous equation as: $2\sin(t)\cos(t) = \cos(t)$ | 1 |
| | Reducing the obtained equation to $\cos(t) = 0 \vee \sin(t) = \frac{1}{2}$ | 1 |
| | $\cos(t) = 0$ gives: $t = \frac{1}{2}\pi,\quad t = 1\frac{1}{2}\pi$<br>$\sin(t) = \frac{1}{2}$ gives: $t = \frac{1}{6}\pi,\quad t = \frac{5}{6}\pi$ | 2 |
| | Giving an argument for the fact that $P$ is above the line $y = 2x$ if $\frac{1}{6}\pi < t < \frac{1}{2}\pi \;\vee\; \frac{5}{6}\pi < t < 1\frac{1}{2}\pi$ | 2 |
| | *Alternative 2:*<br>Rewriting $y = 2x$ as $\sin(2t) + \cos(t) = 2\cos(t)$ | 1 |
| | Rewriting the previous equation as $\cos\left(\frac{\pi}{2} - 2t\right) = \cos(t)$ or as:<br>$\sin(2t) = \sin\left(\frac{\pi}{2} - t\right)$ | 1 |

| | | |
|---|---|---|
| | This yields: $\frac{\pi}{2} - 2t = t \pm k \cdot 2\pi \ \vee\ \frac{\pi}{2} - 2t = -t \pm k \cdot 2\pi$<br>Or: $2t = \frac{\pi}{2} - t \pm k \cdot 2\pi \ \vee\ 2t = \pi - \left(\frac{\pi}{2} - t\right) \pm k \cdot 2\pi$ | 1 |
| | Therefore: $t = \frac{\pi}{6} \pm k \cdot \frac{2}{3}\pi \ \vee\ t = \frac{\pi}{2} \pm k \cdot 2\pi$ | 1 |
| | This gives: $t = \frac{1}{6}\pi,\ t = \frac{5}{6}\pi,\ t = \frac{1}{2}\pi,\ t = 1\frac{1}{2}\pi$ | 1 |
| | Giving an argument for the fact that $P$ is above the line $y = 2x$ if: $\frac{1}{6}\pi < t < \frac{1}{2}\pi \ \vee\ \frac{5}{6}\pi < t < 1\frac{1}{2}\pi$ | 2 |